INDIAN STATISTICAL INSTITUTE
Bangalore center
Mid-Term Examination
February 25, 2019

Topology B.Math II Instructor : Santhosh Kumar P Total : 30 Marks

---

Answer any five questions. Each question carries six marks.

1. Prove the following statements.
   (a) A sequence in $\mathbb{R}$ with cocountable topology is convergent if and only if the sequence is eventually constant. [3]
   (b) Every Hausdorff space is T_1. Is the converse true ? Justify. [3]
2. Define basis for a topological space. Show that (topologically) there are infinitely many prime numbers. [1+5]
3. Let X be a toplogical space. Then show that a bijection $f\colon X \to X$ is a homeomorphism if and only if $f(\overline{A}) = \overline{f(A)}$ for all $A \subseteq X$. [6]
4. Show that every subset of a normal space is completely regular (Hint: use Uryshon's lemma). Is $\mathbb{R}^{\mathbb{R}}$ metrizable ? Justify. [3+3]
5. Provide one example for each of the following with explanation. [3+3]
   (a) A separable first countable space need not to be second countable.
   (b) A subspace of a separable space need not to be separable.
6. State Uryshon's metrization theorem and Tychnoff's theorem. Show that $[0, 1]^{\mathbb{N}}$ equipped with the box toplogy is not compact. [2+4]

ALL THE BEST